14/11/2015 د. سلام [7] محاضرة

$E(x) = \mu = m = \bar{x}$ = 1st moment about origin

$E(x^2)$ 2nd " " origin

$E(x-\mu)^2 = E(x^2) - \mu^2 = Var(x) = \sigma^2$ = 2nd moment about the mean

$\sigma = +\sqrt{\sigma^2} = +\sqrt{Var(x)}$

① $Var(c) = 0$?, $E(c) = E(5) = 5$; $E(c) = c$

$= E(c-m)^2$ ⇓

$= (c-c)^2 = 0$

; $\int x\, p(x)\, dx$

$\int c \cdot p(x)\, dx$

$= c \int p(x)\, dx = c \cdot 1 = c$

② $Var(aX) = a^2\, var(X)$?

⇓

$Var(ax) = E(ax-m)^2 = E[a^2x^2 - 2amX + m^2]$

$= a^2 E(x^2) - E(2amX) + m^2$

$= a^2 E(x^2) - 2am^2 + m^2$

$= \ldots\ldots$

③ $Var(ax+b) = a^2\, Var(x)$

⇓

$Var(ax+b) = Var(ax) + Var(b)$

$= a^2\, Var\, x + 0$

$= a^2\, Var\, X$

⟹ turn over

Example:

$$Var(2X) = 4\,Var(X)$$
$$Var(2X-3) = 4\,Var(X)$$
$$Var(1-X) = Var(X)$$
$$Var(2X+1000) = 4\,Var(X)$$

Prove that $Var(aX) = a^2 Var(X)$: —

$$Var(aX) = E(aX)^2 - m^2$$
$$= E(aX)^2 - [E(aX)]^2 \rightarrow a^2[E^2(X)]$$
$$= a^2(E(X^2) - E^2(X))$$
$$= a^2[E(X^2) - \mu^2] = a^2\,var(X)$$

* There is a physical interpretation of mean and variance. Suppose at each point x_i on the x axis there is placed a unit with mass $p(x_i)$, then the mean is centre of gravity. the system and the variance is the moment of inertia of the system.

Example:-

A pair of fair dice is tossed. Let X assign to the sum of dice's numbers. Then X is a random variable with probabilities as shown, compute the

mean, Variance, S.D. of X
(Standard deviation)

For (3) => another proof

$$Var(aX+b)$$
$$= E[(aX+b) - E(aX+b)]^2$$
$$= E[aX+b - aE(X) - b]^2$$
$$= E[aX - aE(X)]^2$$
$$= a^2 E[X - E(X)]^2$$
$$= a^2 E[X-m]^2$$
$$= a^2 Var(X)$$

مقياس التشتت

$Var(X) = \sigma^2$

$\sigma = +\sqrt{Var(X)}$ = Standard Deviation

S.D.

$Var(X) = E[X-m]^2$

$Var(\alpha(X)) = E[\alpha(X) - E(\alpha(X))]$

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	1	2	3	4	5	6	5	4	3	2	1

$m = \bar{X} = E(x) = \sum x_i P(x_i)$

$= \frac{1}{36}[2\times1 + 3\times2 + 4\times3 + 5\times4 + 6\times5 + 7\times6 + 8\times5 + 9\times4 + 10\times3 + 11\times2 + 12\times1]$

$= 7$

$Variance(X) = E(x^2) - m^2$

$E(x^2) = \sum_{x=2}^{12} x_i^2 P(x_i) = \frac{1}{36}[2\times1^2 + 3\times2^2 + 4\times3^2 + 5\times4^2 + 6\times5^2 + 7\times6^2 + 8\times5^2 + 9\times4^2 + 10\times3^2 + 11\times2^2 + 12\times1^2]$

$= 54.8$

then $Var(X) = 54.8 - 49 = 5.8$

$S.D. = \sqrt{Var(X)} = \sqrt{5.8} = 2.4$

⟹ turn over

remember

$$\binom{9}{3} = \frac{9 * 8 * 7}{3 * 2 * 1}$$

* Discrete distribution :-

1) Binomial Distribution:-

we consider repeated and independent trials of an experiment with two outcomes; we call one of the outcomes success and the other outcome failure. Let p be the probability of success, so that $q = 1 - p$ is the probability of failure. Then the probability of getting exactly k success in n repeated trial is denoted and given by $P(k; n, p)$

$$P(k; n, p) = \binom{n}{k} p^k q^{n-k} \quad \text{for } k = 0, 1, 2, \ldots, n$$

It is called the binomial distribution since for $k = 0, 1, 2, 3, \ldots, n$ it corresponds to the successive terms of the binomial expansion

$$(p+q)^n = p^n + \binom{n}{1} p^{n-1} q + \binom{n}{2} p^{n-2} q^2 + \cdots + q^n$$

The distibution is also called the Bernolli distribution, and independent trials with two outcomes are called Bernoulli trials.

ex: $P(3; 10, 1/2)$

$P(4; 20, 1/3)$

$P(16; 20, 2/3)$